

REPORT 1193

THEORETICAL PERFORMANCE CHARACTERISTICS OF SHARP-LIP INLETS AT SUBSONIC SPEEDS¹

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SUMMARY

A method is presented for the estimation of the subsonic-flight-speed characteristics of sharp-lip inlets applicable to supersonic aircraft. The analysis, based on a simple momentum balance consideration, permits the computation of inlet-pressure-recovery—mass-flow relations and additive-drag coefficients for forward velocities from zero to the speed of sound.

The penalties for operation of a sharp-lip inlet at velocity ratios other than 1.0 may be severe; at lower velocity ratios an additive drag is incurred that is not cancelled by lip suction, while at higher velocity ratios, unavoidable losses in inlet total pressure will result. In particular, at the take-off condition, the total pressure and the mass flow for a choked inlet are only 79 percent of the values ideally attainable with a rounded lip. Experimental data obtained at zero speed with a sharp-lip supersonic inlet model were in substantial agreement with the theoretical results.

INTRODUCTION

Air inlets designed for operation at supersonic speeds generally must employ thin, sharp lips if the large drag penalties associated with blunt lips at these speeds are to be avoided. A turbojet-powered supersonic aircraft must take off and accelerate at subsonic Mach numbers, however; therefore, it is of importance to be able to estimate sharp-lip inlet characteristics in the low-speed range as well as at supersonic velocities.

This report presents a simple method developed at the NACA Lewis laboratory for estimating the zero-angle-of-attack characteristics of sharp-lip inlets at subsonic flight speeds. Total-pressure recoveries and additive-drag coefficients are presented for flight velocities from zero to the speed of sound over the full range of inlet operating conditions.

SYMBOLS

The following symbols are used in this report:

A	flow area
A_x	area projection on plane normal to inlet axis
a	local speed of sound
a_a	stagnation speed of sound, $a \left(1 + \frac{\gamma-1}{2} M^2\right)^{\frac{1}{\gamma}}$
b	external body surface
C_{D_a}	additive-drag coefficient, $\frac{D_a}{q_0 A_1}$
D_a	additive drag
F	lip suction force
M	Mach number, V/a
m	mass flow, $\rho V A = \frac{\gamma p M A}{a}$
m^*	reference mass flow (eq. (5))

P	total pressure, $p \left(1 + \frac{\gamma-1}{2} M^2\right)^{\frac{\gamma}{\gamma-1}}$
p	static pressure
q	dynamic pressure, $\frac{1}{2} \rho V^2 = \frac{\gamma}{2} p M^2$
s, s'	streamlines
V	velocity
Φ	momentum parameter, $mV + (p - p_0)A = \gamma p M^2 A + (p - p_0)A$
γ	ratio of specific heats, 1.4 for air
ρ	mass density

Subscripts:

d	external downstream station
t	throat
0	free stream
1	inlet
2	diffuser outlet

ANALYSIS

DETERMINATION OF INLET MOMENTUM PARAMETER

The inviscid-potential-flow pattern into a cylindrical air inlet operating at subsonic free-stream Mach numbers is shown schematically in figure 1 (a). (The word "cylindrical" does not necessarily imply a circular cross section in this report.) The stagnation point of the dividing streamline s occurs inside of the lip for inlet velocity ratios less than 1.0 (corresponding to $M_1/M_0 < 1$ or $A_0/A_1 < 1$) and outside of the lip for velocity ratios greater than 1.0 ($M_1/M_0 > 1$ or $A_0/A_1 > 1$), as shown in reference 1 for the two-dimensional incompressible case. Two important characteristics of this ideal flow may be mentioned: (1) The total pressure is constant throughout the flow field, and (2) a finite suction force F exists on the lip as indicated by the dashed vectors.

For extremely thin inlet lips, the actual flow will differ substantially from the ideal case. In particular, a zero-thickness lip cannot sustain any suction force, and the flow cannot turn the 180° required to stay attached to the wall. The total pressure of the actual flow will not remain constant in the regions affected by the resultant separation. As indicated in figure 1 (b), for $A_0/A_1 < 1$ the external flow will be separated, while the internal flow will not. The internal flow for this case will be isentropic, with skin friction neglected, and will have a streamline pattern similar to the ideal case. In like manner, for $A_0/A_1 > 1$ the external flow will be similar to the ideal, but the internal flow will be separated with a resultant loss in total pressure. The actual flow phenomena are complex, but one-dimensional approximations to total-pressure recoveries and inlet forces may be determined by a simple momentum balance consideration.

¹ Supersedes NACA TN 3004, "Theoretical Performance Characteristics of Sharp-Lip Inlets at Subsonic Speeds," by Evan A. Fradenburgh and DeMarquis D. Wyatt, 1953.

Inlet velocity ratio greater than 1.0.—For the actual flow into an inlet for velocity ratios greater than 1.0 ($A_0/A_1 > 1$), the inlet conditions will not be uniform but may be approximated by an equivalent one-dimensional flow of the same mass flow, energy, and momentum parameter. With this assumption of one-dimensional flow, the inlet station may be considered to be at any point within the constant-area section behind the lip. The conservation of energy requirement will be satisfied if the total temperature and, consequently, the stagnation speed of sound of the flow is held constant. Calculation of the inlet momentum parameter as a function of mass flow will permit the calculation of all the characteristics of this equivalent flow.

The momentum parameter of the internal flow at the inlet Φ_1 is equal to the free-stream value plus all forces exerted on the internal flow in a downstream direction. These forces, for velocity ratios greater than 1.0, include the lip suction force F and the integral of the pressure increment along the stagnation streamline up to the stagnation point (all pressure forces are referenced to free-stream static pressure).

$$\Phi_1 = \Phi_0 + F + \int_0 (p - p_0) dA_x \quad (A_0/A_1 > 1) \quad (1)$$

The pressure integral in equation (1) may be evaluated by replacing the stagnation streamline for figure 1 (b), $A_0/A_1 > 1$, by a solid boundary and determining the inviscid-potential-flow force on this boundary. This may be done with the aid of the theorem that the drag of any closed body without sharp edges is zero in subsonic inviscid flow. This theorem may easily be extended to show that the drag of a body beginning and ending with cylindrical sections of infinite length parallel to the free stream is also zero (ref. 2, appendix I). With the assumption that the stagnation streamline for figure 1 (b), $A_0/A_1 > 1$, is independent of

downstream disturbances, this modified theorem indicates that the pressure integral in equation (1) must be zero for a cylindrical inlet. A mathematical proof of this fact may be found in the appendix.

The expression for the inlet momentum parameter (eq. (1)) is consequently reduced to

$$\Phi_1 = \Phi_0 + F \quad (A_0/A_1 > 1) \quad (2)$$

For a zero-thickness lip, $F=0$, so that

$$\Phi_1 = \Phi_0 \quad (\text{sharp lip, } A_0/A_1 > 1) \quad (3)$$

Thus for a sharp-lip cylindrical inlet at velocity ratios greater than 1.0, there is no change in the momentum parameter from free stream to the inlet.

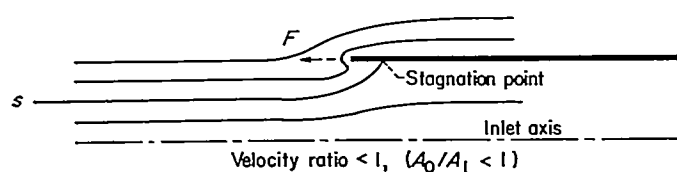
Inlet velocity ratio less than 1.0.—For inlet velocity ratios less than 1.0 ($A_0/A_1 < 1$), the only force exerted on the internal flow between free stream and the stagnation point inside the lip is the pressure integral along the stagnation streamline

$$\Phi_1 = \Phi_0 + \int_0 (p - p_0) dA_x \quad (A_0/A_1 < 1) \quad (4)$$

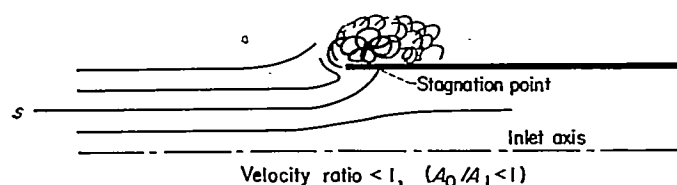
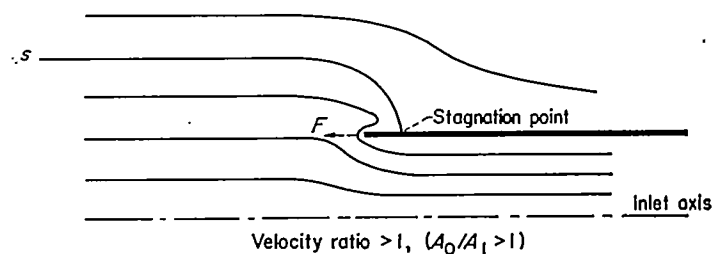
In contrast with the case of velocity ratios greater than 1.0, the pressure integral in equation (4) is not generally zero. The inlet momentum parameter for this case may be determined by the condition that the total pressure of the internal flow is constant. For a given free-stream condition, the inlet Mach number and the total pressure are sufficient to determine the value of the pressure integral in equation (4) and the inlet momentum parameter Φ_1 .

EVALUATION OF SHARP-LIP INLET-PRESSURE-RECOVERY-MASS-FLOW RELATIONS

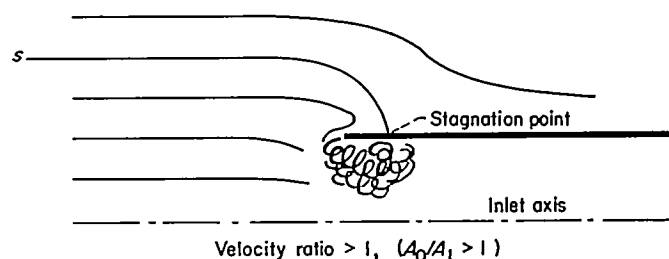
The mass flow through the inlet is



(a)



(b)



(a) Inviscid potential flow.
(b) Actual flow.

FIGURE 1.—Flow patterns for sharp-lip inlet.

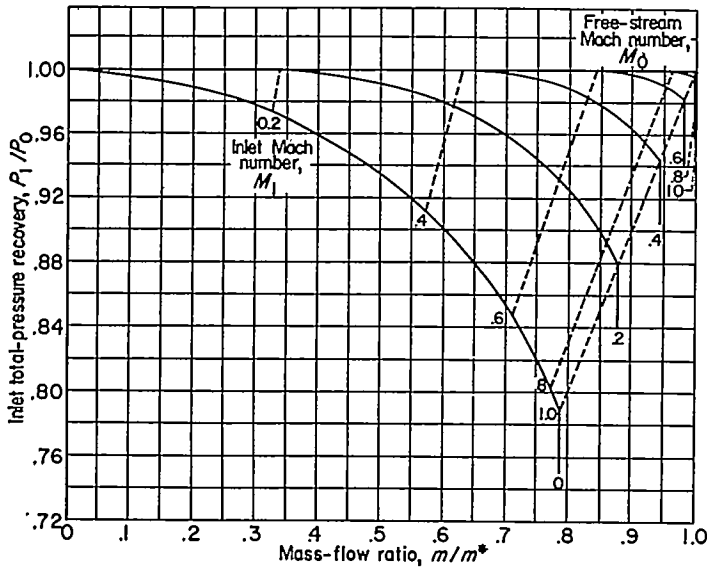


FIGURE 2.—Variation of inlet total-pressure recovery with mass-flow ratio for sharp-lip inlet.

$$m = \rho_1 V_1 A_1 = \frac{\gamma p_1 \left(M \frac{a_a}{a} \right)_1 A_1}{a_a}$$

where

$$\frac{a_a}{a} = \left(1 + \frac{\gamma-1}{2} M^2 \right)^{\frac{1}{2}}$$

A reference mass flow is defined as the value corresponding to choking ($M=1.0$) at the inlet flow area at free-stream total pressure:

$$m^* = \frac{\gamma P_0 \left(\frac{p}{P} \right)_{M=1} \left(M \frac{a_a}{a} \right)_{M=1} A_1}{a_a} \quad (5)$$

where

$$\frac{p}{P} = \left(1 + \frac{\gamma-1}{2} M^2 \right)^{-\frac{\gamma}{\gamma-1}}$$

The mass-flow ratio for $\gamma=1.4$ is then

$$\frac{m}{m^*} = 1.729 \left(\frac{P_1}{P_0} \right) \left(\frac{p}{P} \right)_1 \left(M \frac{a_a}{a} \right)_1 \quad (6)$$

For inlet velocity ratios less than 1.0, the total pressure at the inlet is equal to the free-stream value

$$\frac{P_1}{P_0} = 1.0 \quad (A_0/A_1 < 1) \quad (7)$$

For inlet velocity ratios greater than 1.0, the inlet total pressure is determined by the momentum parameter relation (eq. (3)) and the mass-flow continuity relation. From equation (3),

$$\Phi_1 = \gamma p_1 M_1^2 A_1 + (p_1 - p_0) A_1 = \Phi_0 = \gamma p_0 M_0^2 A_0 \quad (\text{sharp lip}, A_0/A_1 > 1) \quad (8)$$

The continuity relation may be written

$$p_1 \left(M \frac{a_a}{a} \right)_1 A_1 = p_0 \left(M \frac{a_a}{a} \right)_0 A_0 \quad (9)$$

Combining equations (8) and (9) and using the relation $\frac{P_1}{P_0} = \frac{p_1}{p_0} \frac{(p/P)_0}{(p/P)_1}$ yields the following expression for the inlet total-pressure ratio:

$$\frac{P_1}{P_0} = \frac{(p/P)_0}{(p/P)_1} \left[\frac{\gamma M_1^2 + 1 - \gamma M_0^2 \left(M \frac{a_a}{a} \right)_1}{\left(M \frac{a_a}{a} \right)_0} \right] \quad (\text{sharp lip}, A_0/A_1 > 1) \quad (10)$$

The inlet total-pressure recovery is thus a function only of the free-stream and inlet Mach numbers. Because the pressure recovery and the inlet Mach number determine the mass-flow ratio (eq. (6)), the pressure-recovery—mass-flow relation is uniquely determined by the free-stream and inlet Mach numbers. This relation (eqs. (6), (7), and (10)) is plotted in figure 2 for free-stream Mach numbers from zero (corresponding to static or take-off condition) to 1.0. At an inlet Mach number M_1 equal to 1.0, decreasing the diffuser-outlet pressure will not increase the mass flow but will result only in supersonic flow in the divergent part of the diffuser, with resultant additional pressure losses, so that the curves drop off vertically from this point.

The penalties for operating a sharp-lip inlet near choking are severe for low free-stream Mach numbers. At zero forward speed, the total-pressure recovery and the mass-flow ratio are each 0.79 for a choked inlet, compared with 1.0 ideally attainable with an inlet of large lip radius.

The relation between the mass-flow ratio as defined and the flow area ratio may be determined by combining equations (6) and (9):

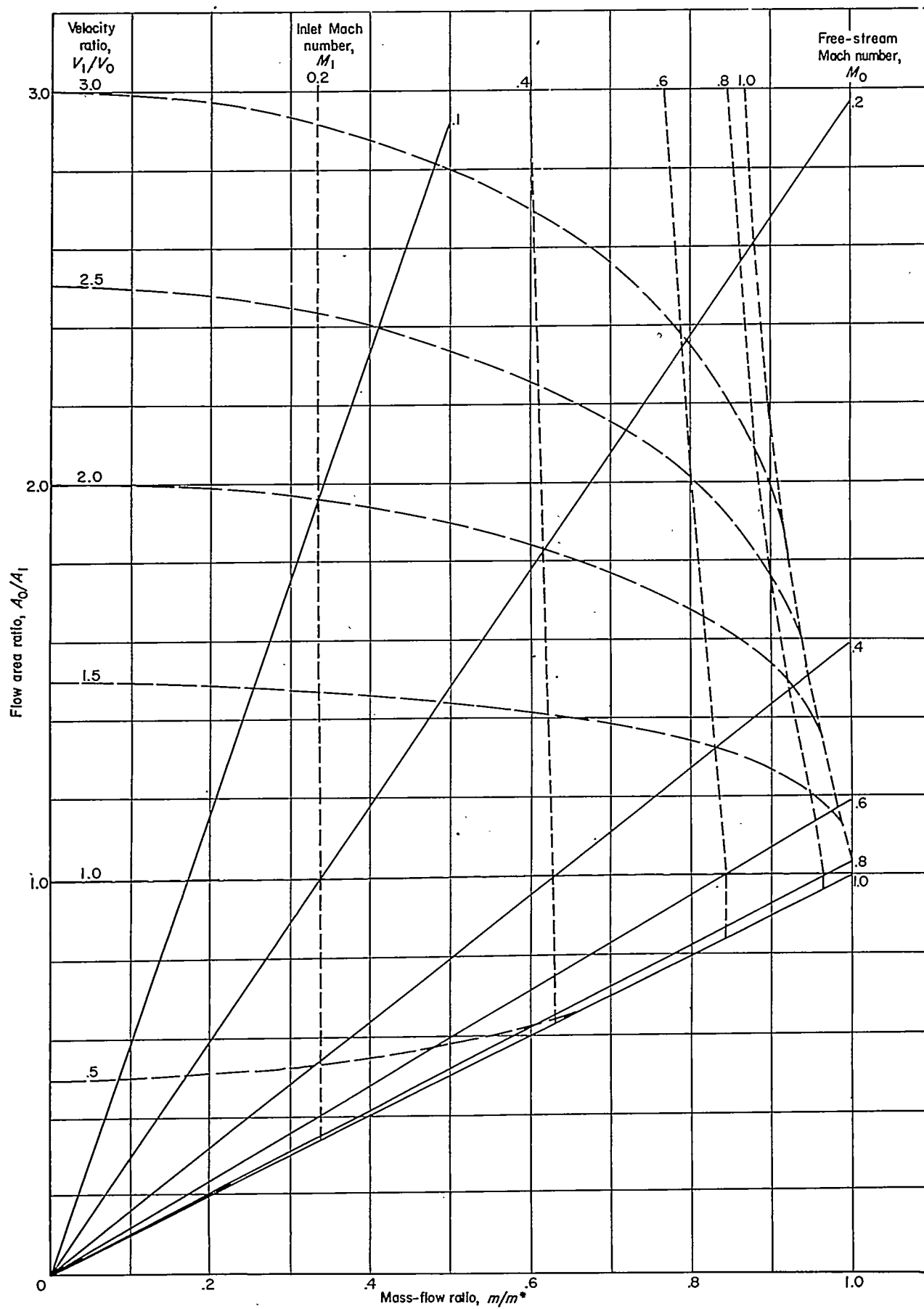


FIGURE 3.—Variation of flow parameters with mass-flow ratio for sharp-lip inlet.

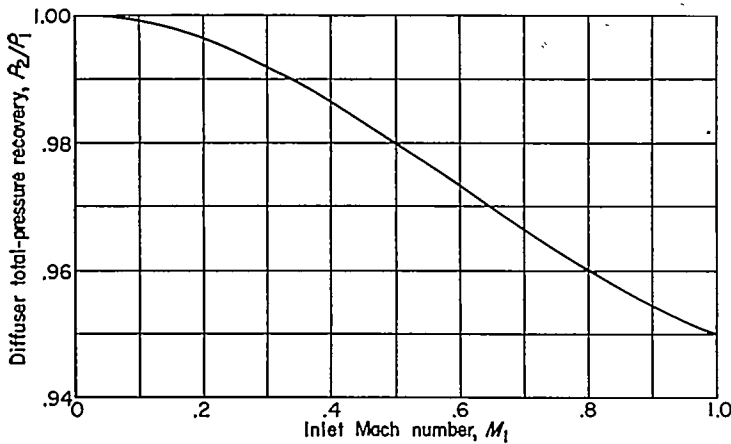


FIGURE 4.—Estimated diffuser total-pressure recovery.

$$\frac{A_0}{A_1} = 0.578 \frac{m/m^*}{(p/P)_0 \left(M \frac{a}{a_0} \right)} \quad (11)$$

This equation is plotted for convenience in figure 3 for a range of free-stream Mach numbers. For $M_0=0$, this area ratio is infinite for all finite mass flows. Also included in this figure for reference purposes are lines of constant inlet Mach number M_1 and lines of constant velocity ratio $\frac{V_1}{V_0} = \frac{M_1 a_1}{M_0 a_0}$, both corresponding to the sharp-lip case.

The only pressure losses discussed so far are those necessitated by the inlet momentum consideration. According to the one-dimensional approximation, these losses must occur in some manner ahead of the inlet. Presumably the actual losses occur throughout the inlet region, mainly by the mechanism of turbulent mixing. Additional losses can be expected to occur in the diffuser behind the inlet and must be considered in an over-all performance evaluation. These losses may be approximated by assuming that the decrease in total pressure from the inlet to the diffuser outlet is proportional to the inlet dynamic pressure:

$$P_1 - P_2 = k q_1 = k \frac{\gamma}{2} p_1 M_1^2$$

The value of k selected for a well-designed diffuser is 0.135, which corresponds to a 5-percent total-pressure loss for $M_1=1.0$. The resultant variation of diffuser total-pressure ratio with inlet Mach number is plotted in figure 4. This estimated diffuser pressure recovery is combined with the inlet recovery (fig. 2) and the resultant over-all pressure recovery is plotted against the mass-flow ratio in figure 5.

Also presented in figure 5 are some experimental data obtained in quiescent-air tests of a sharp-lip inlet designed for supersonic speeds. This inlet, which is sketched in figure 5, was a semicircular scoop mounted on a flat plate, with a 25° half-angle-cone centerbody. The external lip angle measured from the inlet centerline was approximately 20°. This model, although considerably different from the cylindrical inlet assumed in the analysis, gave results quite similar to the theoretical zero-speed variation. The maximum mass flow measured was in excellent agreement with the theoretical value. The experimental pressure recoveries were somewhat lower than the estimated values, which

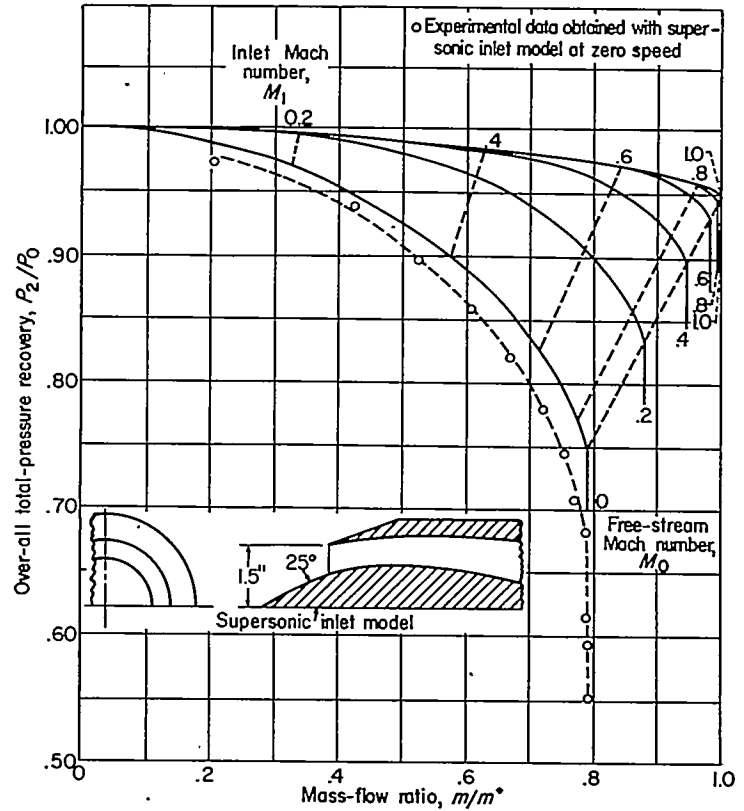


FIGURE 5.—Estimated over-all total-pressure recovery for sharp-lip inlet.

suggests that the diffuser losses were higher for this model than those assumed. Data have not been obtained for this model at finite subsonic forward speeds.

ADDITIVE DRAG

The thrust of a jet-engine installation is conventionally defined as the outlet momentum parameter minus the free-stream momentum of the air passing through the propulsive duct. When the inlet momentum parameter is not equal to the free-stream momentum, the difference between the two values must be added to the external drag of the aircraft to make the resultant thrust-minus-drag equal to the actual net force. In the ideal case of a rounded inlet lip operating at subsonic speeds, the lip suction force just cancels this additive drag, so that the sum of these two forces may be neglected. No cancellation will occur, however, if the inlet lip is extremely thin and sharp. With the additive drag D_a defined as $\Phi_1 - \Phi_0$, the expression for the additive-drag coefficient based on the inlet area is

$$C_{D_a} = \frac{D_a}{q_0 A_1} = \frac{\Phi_1 - \Phi_0}{\gamma \frac{p_0}{2} M_0^2 A_1} = \frac{2}{\gamma M_0^2} \left(\gamma M_1^2 \frac{p_1}{p_0} + \frac{p_1}{p_0} - 1 \right) - 2 \frac{A_0}{A_1} \quad (12)$$

For a sharp-lip inlet operating at velocity ratios greater than 1.0, from equations (3) and (12),

$$C_{D_a} = 0 \text{ (sharp lip, } A_0/A_1 > 1) \quad (13)$$

For velocity ratios less than 1.0, the additive drag is evaluated by the condition that $P_1 = P_0$ (eq. (7)). This relation, combined with equations (9) and (12), yields the following expression for the additive-drag coefficient:

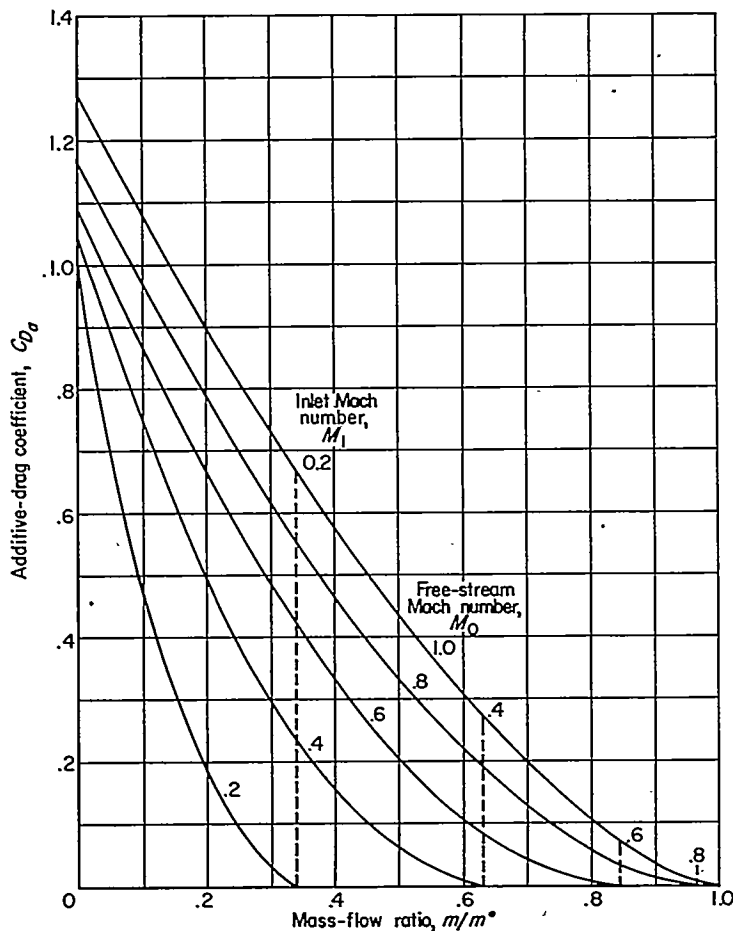


FIGURE 6.—Additive-drag coefficient for sharp-lip inlet.

$$C_{D_a} = \frac{2}{\gamma M_0^2} \left(\gamma M_1^2 \frac{(p/P)_1}{(p/P)_0} - 1 \right) - 2 \frac{(p/P)_1}{(p/P)_0} \frac{\left(M \frac{a}{a} \right)_1}{\left(M \frac{a}{a} \right)_0} \quad (A_0/A_1 < 1) \quad (14)$$

The additive-drag coefficient is thus a function only of M_0 and M_1 . A plot of C_{D_a} (eqs. (13) and (14)) against mass-flow ratio is presented in figure 6. It should be pointed out that the values shown for velocity ratios less than 1.0 ($M_1 < M_0$) are equal to the net inlet drag only for a zero-thickness lip. For any finite thickness some cancellation of this drag due to lip suction will occur, and in the ideal case the theoretical lip suction (ref. 1) is exactly equal to the additive drag. For velocity ratios greater than 1.0, the net inlet drag will be zero for both sharp and rounded lips.

The additive-drag coefficient is highest at low mass-flow ratios and high free-stream Mach numbers. Operation of a sharp-lip inlet at velocity ratios greater than 1.0 avoids the additive drag but results in inlet total-pressure losses (fig. 2). Evidently a velocity ratio of 1.0 is the only condition for a sharp-lip inlet that avoids both additive-drag and pressure-recovery penalties. Ideally, a well-rounded lip permits operation at any velocity ratio without penalty.

EFFECT OF INTERNAL CONTRACTION

In some supersonic-inlet designs, a contraction in flow area is placed behind the inlet to reduce the supersonic Mach

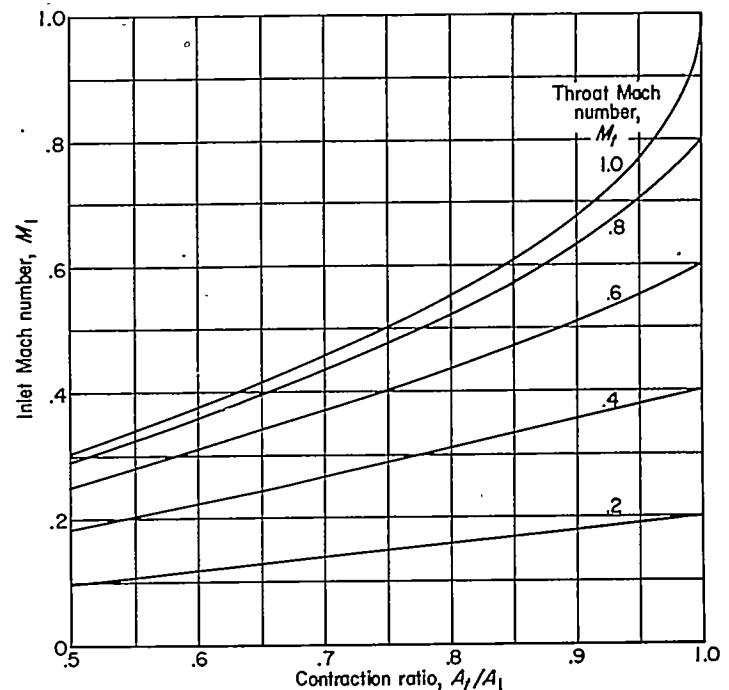


FIGURE 7.—Effect of contraction ratio on inlet Mach number

number before the terminal shock occurs, thereby reducing the shock losses. In order to estimate the effect of internal contraction on inlet performance at subsonic speeds, it is assumed that isentropic flow occurs between the inlet and the minimum area or throat. The mass-flow continuity relation may be written

$$p_1 \left(M \frac{a}{a} \right)_1 A_1 = p_t \left(M \frac{a}{a} \right)_t A_t$$

or, since P_t is assumed equal to P_1 ,

$$(p/P)_1 \left(M \frac{a}{a} \right)_1 = (p/P)_t \left(M \frac{a}{a} \right)_t \frac{A_t}{A_1} \quad (15)$$

With the use of equation (15), the inlet Mach number is plotted as a function of the contraction ratio A_1/A_1 and the throat Mach number in figure 7. Total-pressure-recovery-mass-flow-ratio characteristics may be estimated for any value of inlet contraction ratio by finding the inlet Mach number M_1 as a function of throat Mach number from this figure. The inlet Mach number thus determined and the free-stream Mach number may then be used directly to determine the inlet total-pressure recovery, the mass-flow ratio, and the additive drag from figures 2 and 6. The subsonic-diffuser losses in this case may be estimated approximately from figure 4 by using the throat Mach number M_t rather than M_1 as the abscissa.

In order to illustrate some of the effects of contraction ratio, the inlet total-pressure recovery, mass-flow ratio, and additive-drag coefficient are shown as a function of the contraction ratio in figure 8 for critical flow conditions (choked at throat, $M_t = 1.0$). The inlet pressure recovery P_1/P_0 is

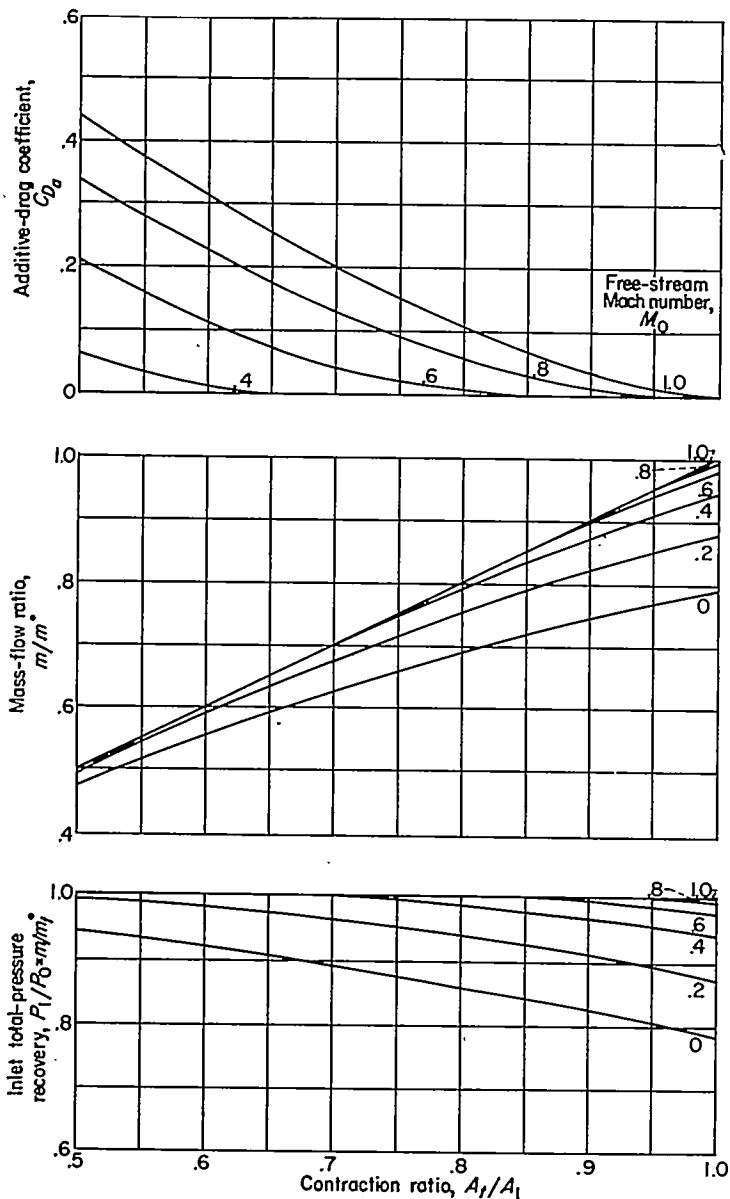
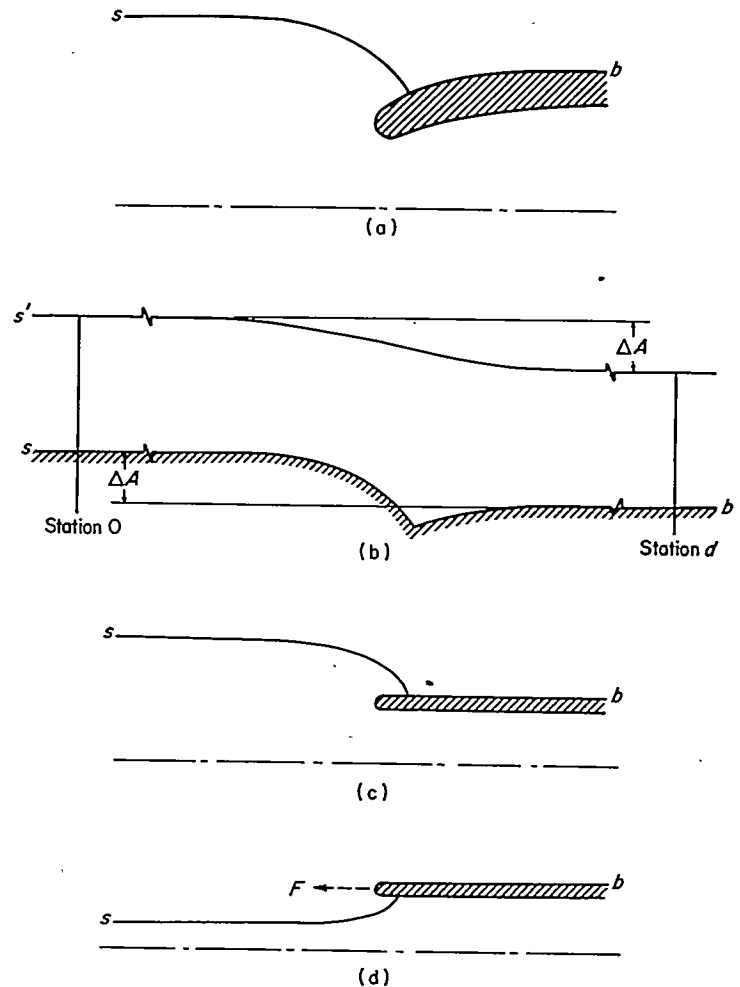


FIGURE 8.—Effect of contraction ratio on sharp-lip inlet performance at critical flow (throat Mach number, 1.0).

increased by a contraction for the lower free-stream Mach numbers because of the reduction in inlet Mach number. A mass-flow ratio m/m^* , where m^* corresponds to isentropic choking at the throat area A_t rather than the inlet area A_1 , is equal to P_1/P_0 for this case. Thus the mass flow for a given minimum flow area A_t increases as the ratio of throat area to inlet area decreases. The mass-flow ratio based on choking at the inlet area m/m^* , however, decreases as A_t/A_1 decreases. It may also be seen that an internal contraction carries an additive-drag penalty at the higher free-stream Mach numbers, because the inlet velocity ratio becomes less than 1.0.

CONCLUDING REMARKS

It has been shown that the subsonic-flight-speed characteristics of sharp-lip air inlets applicable to supersonic aircraft



- (a) Curved cowl. Velocity ratio greater than 1.0.
 (b) Equivalent solid boundary.
 (c) Cylindrical cowl. Velocity ratio greater than 1.0.
 (d) Cylindrical cowl. Velocity ratio less than 1.0.

FIGURE 9.—Inlet flow patterns.

may be estimated by a simple momentum balance consideration. Pressure-recovery—mass-flow relations and additive-drag coefficients may be calculated for flight velocities from zero to the speed of sound over the full range of inlet operating conditions.

The penalties for operation at inlet velocity ratios other than 1.0 may be severe; at lower velocity ratios an additive drag is incurred that is not cancelled by lip suction, while at higher velocity ratios, unavoidable losses in inlet total pressure will result. In particular, at zero forward velocity (take-off condition), the total-pressure recovery and the mass-flow ratio for a choked inlet are only 79 percent of the values ideally attainable with a rounded lip. Experimental data obtained at zero speed with a sharp-lip supersonic inlet model were in substantial agreement with the theoretical results.

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NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

CLEVELAND, OHIO, July 27, 1953

APPENDIX

EVALUATION OF PRESSURE INTEGRAL ON STAGNATION STREAMLINE FOR INLET VELOCITY RATIOS GREATER THAN 1.0

The flow into an inlet for velocity ratios greater than 1.0 is represented schematically by figure 9 (a). The stagnation streamline is labeled s and the exterior body surface downstream of the stagnation point is labeled b . The external flow may be considered independently of the internal flow by replacing the stagnation streamline by a solid boundary, as in figure 9 (b), and finding the inviscid-potential-flow solution for the pressure integral of this boundary. It is assumed that the inlet region is connected to the rest of the aircraft by a cylindrical section of sufficient length to make the flow near the inlet independent of disturbances caused by other components of the aircraft. Thus in figure 9 (b) the solid boundary may be assumed to be extended to infinity in both directions without changing the flow near the inlet.

The static pressure at the downstream infinity station d will be equal to the free-stream static pressure. The total pressure is assumed constant, and thus the Mach number at d will be equal to the free-stream Mach number:

$$\begin{aligned} p_d &= p_0 \\ p_d &= P_0 \\ M_d &= M_0 \end{aligned} \quad (A1)$$

When the flow between the solid boundary (s and b) and a streamline s' is considered, the mass flow must be equal at the two stations:

$$\begin{aligned} m_d &= \rho_d A_d V_d = \frac{\gamma \rho_d A_d M_d \left(1 + \frac{\gamma-1}{2} M_d^2\right)^{\frac{1}{2}}}{a_d} = \\ &= \frac{\gamma p_0 A_0 M_0 \left(1 + \frac{\gamma-1}{2} M_0^2\right)^{\frac{1}{2}}}{a_d} = m_0 \end{aligned} \quad (A2)$$

From equations (A1) and (A2), it is evident that the flow areas at the two stations must be equal:

$$A_d = A_0 \quad (A3)$$

The difference in momentum parameter at the two stations is therefore

$$\Phi_d - \Phi_0 = \gamma p_d M_d^2 A_d + (p_d - p_0) A_d - \gamma p_0 M_0^2 A_0 = 0 \quad (A4)$$

As there is no change in the momentum parameter between the two stations, the combined longitudinal force on s , b , and s' must be zero. By selecting a streamline s' a sufficient distance from s , the difference between the static pressure on s' and the free-stream static pressure may be made to approach zero. Since the longitudinal area projection ΔA of this streamline is finite, the longitudinal force on s' must be zero. Thus the combined longitudinal force on s and b must also be zero:

$$\int_s (p - p_0) dA_x + \int_b (p - p_0) dA_x = 0 \quad (A5)$$

For a cylindrical cowl (fig. 9 (c)), the body surface downstream of the stagnation point has no longitudinal area projection as long as the stagnation point of the flow occurs on the external cylindrical portion. Thus for this case, the longitudinal force on s is zero:

$$\int_s (p - p_0) dA_x = 0 \quad (A6)$$

The preceding analysis may be extended to the case of inlet velocity ratios less than 1.0. Figure 9 (d) represents this case for a cylindrical cowl. Equation (A5) indicates that any pressure force on the streamline s is cancelled by the lip suction force F . It should be noted, however, that this result is dependent on the assumption of constant total pressure for the external flow. If the inlet lip is thin, the external flow will separate at the lip and the total pressure will not be constant; therefore, the proof does not apply. The pressure force on the streamline s will, in general, be only partially cancelled by lip suction for this case.

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